University of California, Berkeley — **Economics 230a**
Department of Economics — **Fall 2013**

Problem Set #1

(due 10/8/13)

1. Consider an economy in which relative producer prices are fixed and a representative household, with a unit endowment of labor, maximizes the following utility function:

$$U(c_1, c_2, l) = (c_1 - a_1)^{\beta_1}(c_2 - a_2)^{\beta_2} l^{1-\beta_1-\beta_2}$$

(where c_1 and c_2 are consumption goods and l is leisure), subject to the budget constraint:

$$p_1 c_1 + p_2 c_2 + wl = w$$

A. Derive an explicit solution (i.e., in terms of prices and preference terms a_i and β_i) for the excess burden of taxes on c_1, c_2, and l as a function of the original, undistorted prices of the three goods (p_1^0, p_2^0, and w^0), the distorted prices (p_1^1, p_2^1, and w^1) and a fixed utility level.

B. Show that excess burden equals zero if $p_i^1 = (1+\theta)p_i^0, i$ = 1,2, and $w^1 = (1+\theta)w^0$ for some constant θ.

C. Compare the values of excess burden based on utility levels achieved in the absence and the presence of taxation, $V(p_1^0, p_2^0, w^0)$ and $V(p_1^1, p_2^1, w^1)$.

D. Using the measure derived in part A, show that excess burden for any tax *or* subsidy on good 2 is positive. (*Hint*: show that *marginal* excess burden has the same sign as $(p_2^1 - p_2^0)$.)

2. Consider a model of household production, in which the representative household maximizes the utility of market goods *X* and home goods *Z*, *U*(*X*,*Z*). The household has one source of income, labor, and derives no utility directly from leisure. It supplies some of its unit labor endowment to the market and uses the rest in home production of *Z*. Labor supplied to the market goes into the production either of *X* or an intermediate good, hired day care services, *M*. *X* and *M* are each produced in the market subject to constant returns to scale using labor, and the household produces *Z* subject to constant returns to scale using two factors of production: the labor it withholds from the market, *h*, and hired day care services, *M*.

A. Write down the household's utility and budget constraint (without taxes) as functions of *X*, *M*, and *h*, and show that the problem is equivalent to one in which *X* and *M* are consumption goods with fixed producer prices and *h* is leisure.

B. Suppose that the government has imposed a proportional tax on labor income to raise revenue. It is suggested that efficiency might be enhanced by adding a subsidy to market day care services, *M*, in order to encourage individuals to work. Show that this will be true if and only if the cross-elasticity of demand for *X* with respect to *M*, ε_{XM}, is lower than the cross-elasticity of demand for home labor, h, with respect to M, ε_{hM}. (*Hint*: consider the equivalent tax scheme involving taxes on X and M and derive a condition for these taxes to be equal, using the properties of the utility function derived in part A.)

3. In the Harberger two-sector model, we say that capital bears all of the corporate (sector X) income tax if labor's share of total *before-tax* income is unchanged, that is, if the following ratio stays fixed, as T_{KX} changes:

$$\frac{wL}{wL + rK + (T_{KX} - 1)rK_X}$$

A. For K and L fixed and starting with no taxes (i.e., with T_{KX} initially equal to 1), show that this condition will be satisfied only if the following relationship holds for the relative changes in w and r, where $\lambda_{KX} = K_X/K$:

$$(*) \quad \hat{w} - \hat{r} = \lambda_{KX}\hat{T}_{KX}$$

B. Using the expression derived in class relating $\hat{w} - \hat{r}$ to $\hat{T}_{KX}$, show that (*) holds if X and Y have the same initial factor proportions ($\lambda_{KX} = \lambda_{LX}$) and the same production elasticities of substitution ($\sigma_X = \sigma_Y$).

C. Now suppose that X and Y have the same initial factor proportions but that $\sigma_Y = 0$. Show that capital bears more than 100% of the tax.